Basics of protoplanetary disc structure and dynamics

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Your participation is needed

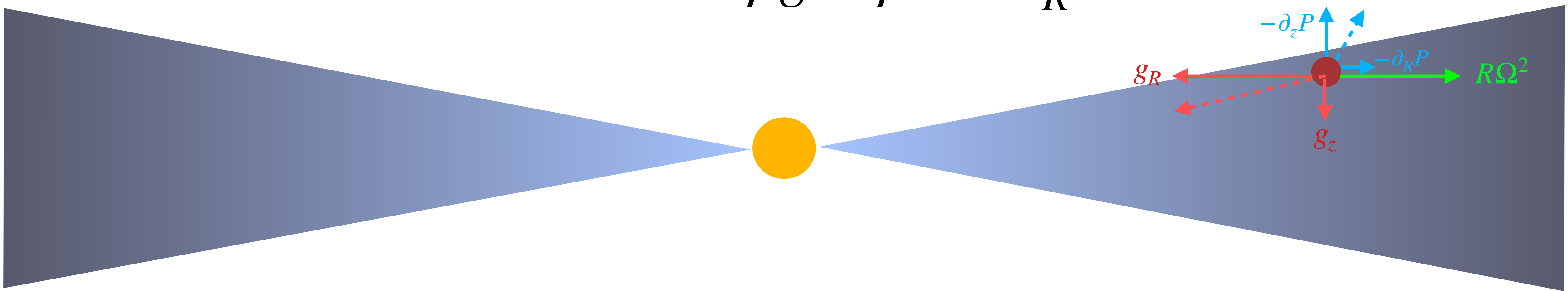
Each kouign amann hidden in the slides is an opportunity to get one of these speciality from Brittany for free if you answer (correctly!) the question

Basic disc structure

- Disc equilibrium
  - Radial equilibrium
  - Vertical equilibrium
- Disc secular dynamics
  - Mass and angular momentum conservation
  - The alpha disc prescription
  - Some alpha disc solutions

Global disc equilibrium

$$0 = -\vec{\nabla} P + \rho \vec{g} + \rho R \Omega^2 \vec{e}_R$$



by components:

$$0 = -\frac{1}{\rho}\partial_R P + g_R + R\Omega^2$$

$$0 = -\frac{1}{\rho}\partial_z P + g_z$$

Open question: should $\boldsymbol{\Omega}$ depend on R? on z?

Constrains on the rotation profile

The gravitational field derives from a potential $\vec{g} = -\nabla\psi$

∂_z $$0 = -\frac{1}{\rho}\partial_R P - \partial_R \psi + R\Omega^2$$

-

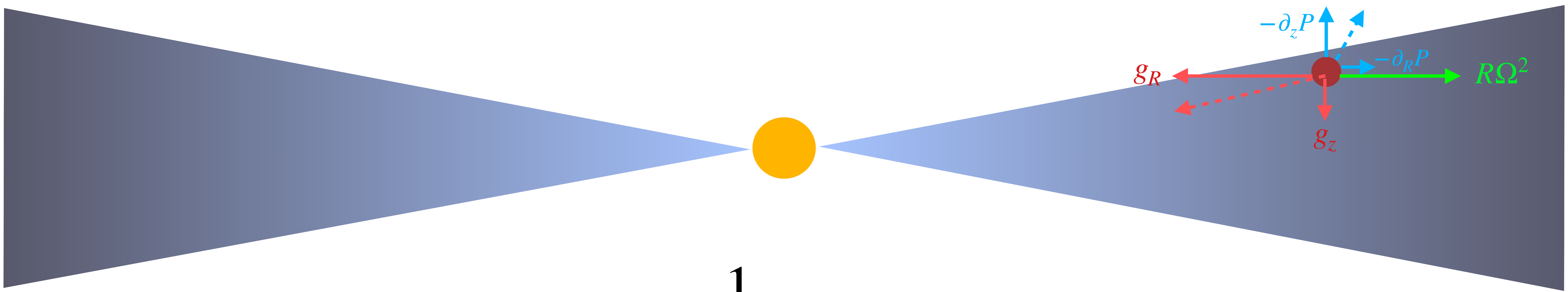
∂_R $$0 = -\frac{1}{\rho}\partial_z P - \partial_z \psi$$

$$\frac{1}{\rho^2}(\partial_z \rho \partial_R P - \partial_R \rho \partial_z P) + R\partial_z \Omega^2 = 0$$

« Thermal wind equation »

- Unless under very specific circumstances (eg Barotropic flow), the rotation profile must depend on z
- This « vertical shear » is driven by the thermal+density disc structure
- It is too often forgotten…

Vertical disc equilibrium



$$0 = -\frac{1}{\rho}\partial_z P + g_z$$

$$g_z = -\frac{GM_\star z}{(R^2+z^2)^{3/2}}$$

Define the isothermal sound speed $c_s^2 \equiv P/\rho \propto T$

Assume the disc is locally-isothermal, i.e $T = T(R)$ and $\partial_z T = \partial_z c_s^2 = 0$

$$\frac{\partial_z \rho}{\rho} = -\frac{GM_\star z}{c_s^2 (R^2+z^2)^{3/2}} \quad \Rightarrow \quad \rho = \rho_{\rm mid}(R)\exp\left[\frac{R^2}{H^2}\left(\frac{R}{(R^2+z^2)^{1/2}} - 1\right)\right]$$

$$H^2 \equiv c_s^2 R^3 / GM_*$$

in the limit $z \ll R$: $\rho = \rho_{\rm mid}(R)\exp\left(-z^2/(2H^2)\right)$

Radial equilibrium

$$0 = -\frac{1}{\rho}\partial_R(\rho c_s^2) + g_R + R\Omega^2$$

- 1 equation, 3 unknowns : $\rho_{\mathrm{mid}}(R), \Omega(R, z), c_s^2(R) \propto T(R)$,
  - $\rho_{\mathrm{mid}}(R)$ will be constrained by the disc temporal evolution
  - $T(R)$ will be constrained by radiative equilibrium
- For now, **we assume** a density and temperature profile:

$$\rho_{\mathrm{mid}}(R) = \rho_0 \left(\frac{R}{R_0}\right)^p \qquad T(R) = T_0 \left(\frac{R}{R_0}\right)^q$$

Putting it all together

assuming $\rho_{\mathrm{mid}}(R) = \rho_0 \left(\frac{R}{R_0}\right)^p$ $T(R) = T_0 \left(\frac{R}{R_0}\right)^q$

vertical equilibrium:

$$\rho(R,z) = \rho_0 \left(\frac{R}{R_0}\right)^p \exp\left[\frac{R^2}{H^2}\left(\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{R}\right)\right]$$

$$\sim \exp\left(-\frac{z^2}{2H^2}\right)$$

Radial equilibrium:

$$\Omega(R,z) = \Omega_K \left[1 + (p+q)\left(\frac{H}{R}\right)^2 + q\left(1 - \frac{R}{\sqrt{R^2+z^2}}\right)\right]$$

$$\Omega_K \equiv \sqrt{\frac{GM_\star}{R^3}}$$

« radial pressure support »

« vertical shear »

[Nelson+ 2013]

Disc structure example

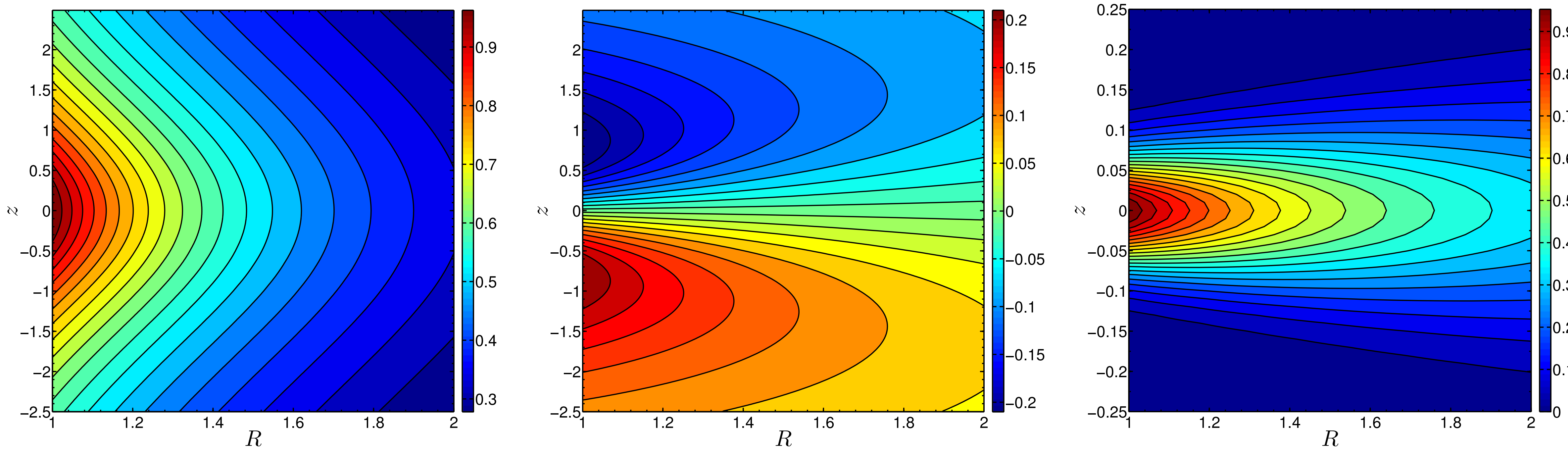


Figure 1. Basic state for the locally isothermal disc with $q = -1$, $p = -1.5$ and $c_0 = 0.05$. The left-hand panel shows a contour plot of Ω on the (R, z) plane. The middle panel is a similar contour plot, but this shows the magnitude of the vertical shear $\partial_z(R\Omega)$, which has a maximum at $|z| \sim 1$ (whereas the scaleheight at the inner radial boundary is 0.05). The right-hand panel shows the density ρ.

[Barker & Latter 2015]

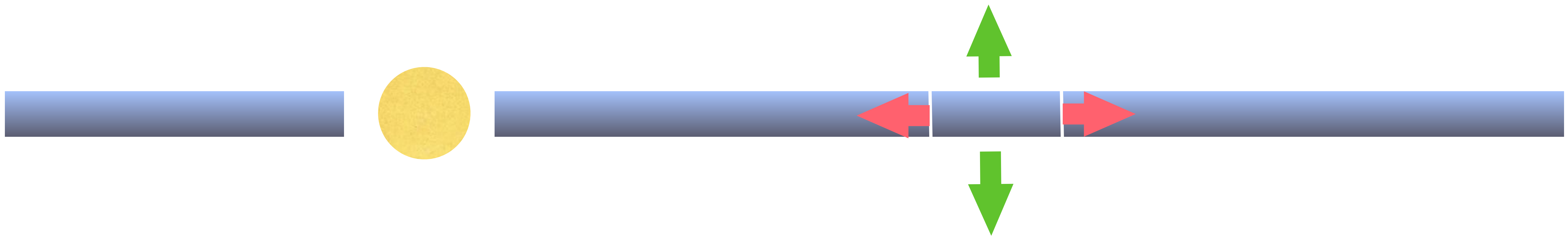
Disque secular dynamics

Disc Dynamics
Mass conservation

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \rho \boldsymbol{u} = 0$$

Introduce the average: $\overline{Q} = \int d\phi \int_{z=-h}^{z=+h} dz\, Q$ and $\Sigma = \overline{\rho}$

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R} R\overline{\rho v_r} + \left[\rho v_z\right]_{z=-h}^{+h} = 0$$



Disc Dynamics

Angular momentum conservation

$$\frac{\partial(\rho R u_\phi)}{\partial t} + \vec{\nabla} \cdot \left[\rho R u_\phi \vec{u} + RP\vec{e}_\phi\right] = 0$$

- Introduce $\vec{u} = \Omega_K \vec{e}_\phi + \vec{v}$:

$$\Omega_K R^2 \frac{\partial \rho}{\partial t} + \frac{\partial(\rho R v_\phi)}{\partial t} + \vec{\nabla} \cdot \left[\rho R^2 \Omega_K \vec{v} + \rho R v_\phi \vec{v} + RP\vec{e}_\phi\right] = 0$$

- Average and integrate vertically:

$$\Omega_K R^2 \frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R} R\left(R^2 \Omega_K \overline{\rho v_r} + R\overline{\rho v_\phi v_r}\right) + \left[R^2 \Omega_K \rho v_z + R \rho v_\phi v_z\right]_{z=\pm h} = 0$$

Cancel with mass conservation

Disc Dynamics

Angular momentum conservation

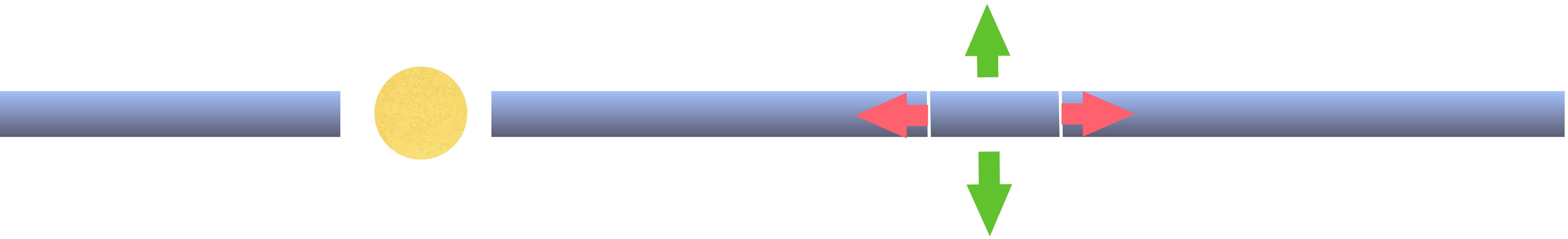
Angular momentum conservation (once mass conservation is taken into account)

$$\overline{\rho v_r}\frac{\partial}{\partial R}\Omega_K R^2 + \frac{1}{R}\frac{\partial}{\partial R}R^2\left[\overline{\rho v_\phi v_r}\right] + R\left[\rho v_\phi v_z\right]_{z=\pm h} = 0$$

accretion

radial stress

vertical stress (aka wind stress)



Disc Dynamics

α disc model

[Shakura & Sunyaev 1973, Lynden-Bell & Pringle 1974]

$$\overline{\rho v_r}\frac{\partial}{\partial R}\Omega_K R^2 + \frac{1}{R}\frac{\partial}{\partial R}R^2\left[\overline{\rho v_\phi v_r}\right] + R\left[\rho v_\phi v_z\right]_{z=\pm h} = 0$$

$$= \alpha\overline{P}$$

- The « α disk » model is a closure for a turbulent disk

$$\overline{\rho\delta v^2} = \alpha\overline{P} = \alpha c_s^2\overline{\rho} \text{ so that } \delta v \simeq \sqrt{\alpha}c_s$$

- The « α disk » model is equivalent to an « effective » viscosity:
  - alpha stress: $\alpha\overline{P} = \alpha c_s^2\overline{\rho} = \alpha c_s H\Omega_K\overline{\rho}$
  - viscous stress: $-\nu R\frac{\mathrm{d}\Omega_K}{\mathrm{d}R}\overline{\rho} = \frac{3}{2}\nu\Omega_K\overline{\rho}$

$$\rightarrow \nu_{\mathrm{eff}} = \frac{2}{3}\alpha c_s H$$

Disc Dynamics

α disc model (cont'd)

[Shakura & Sunyaev 1973, Lynden-Bell & Pringle 1974]

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R} R\overline{\rho v_R} = 0$$

$$\overline{\rho v_r}\frac{\partial}{\partial R}\Omega_K R^2 + \frac{1}{R}\frac{\partial}{\partial R} R^2 \alpha c_s^2 \Sigma = 0$$

$$\Rightarrow \quad \frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}\frac{2}{\Omega_K R}\frac{\partial}{\partial R} R^2 \alpha c_s^2 \Sigma = 0$$

- Effectively a diffusion equation for the surface density

  $\partial_r \Sigma \simeq -\kappa \partial_R^2 \Sigma$ with diffusion coefficient $\kappa = \alpha c_s^2 / \Omega_K$

- Gives a typical timescale for the disk dissipation $\tau = R^2/\kappa = R^2 \Omega_K / \alpha c_s^2$
  - with T=10K (cs=100 m/s), R=100 AU around a solar mass star: $\tau = 40\,000/\alpha$ years
  - Assuming a typical survival timescale ~ a few million years $\alpha \simeq 10^{-2}$

Viscous solutions

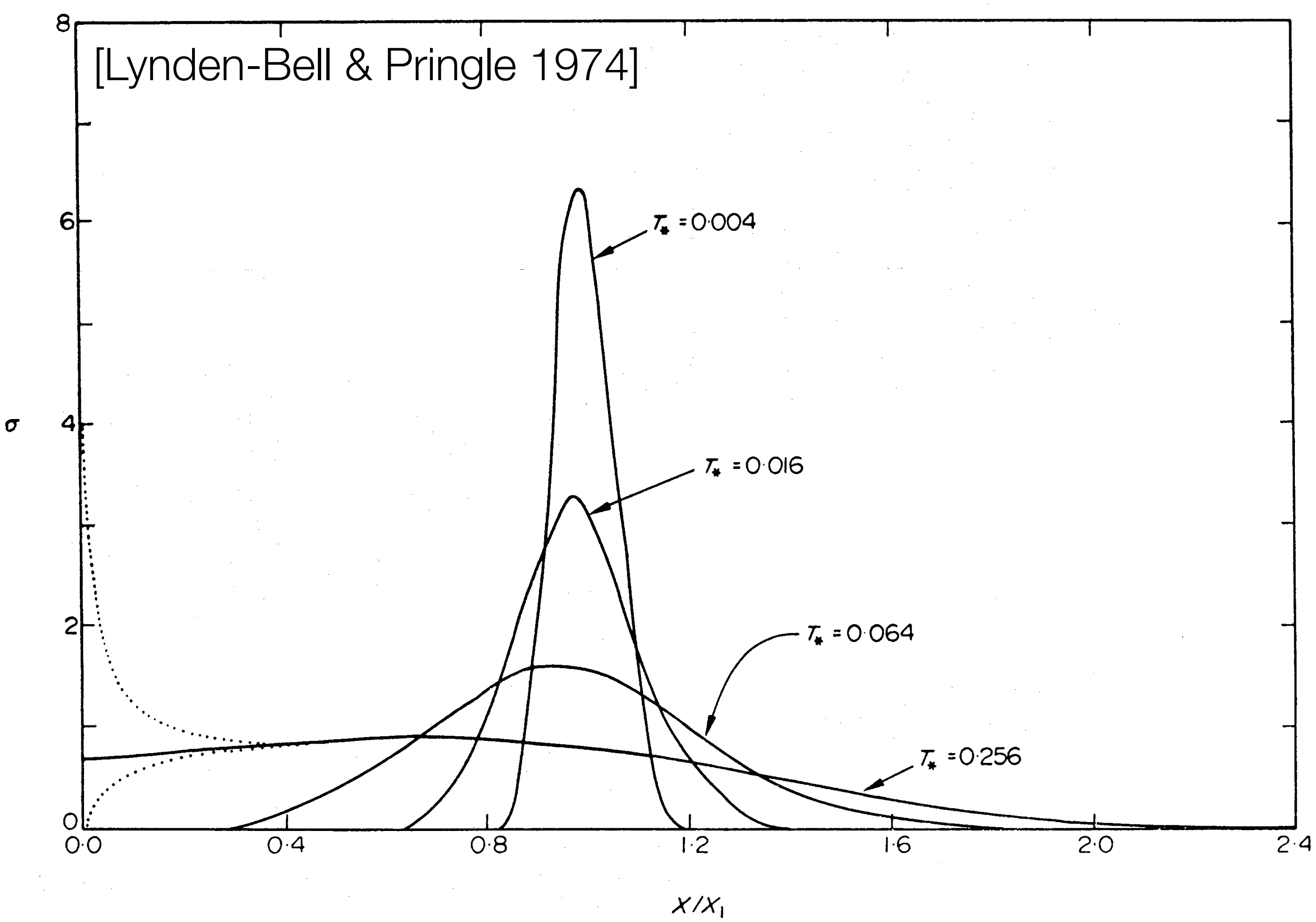
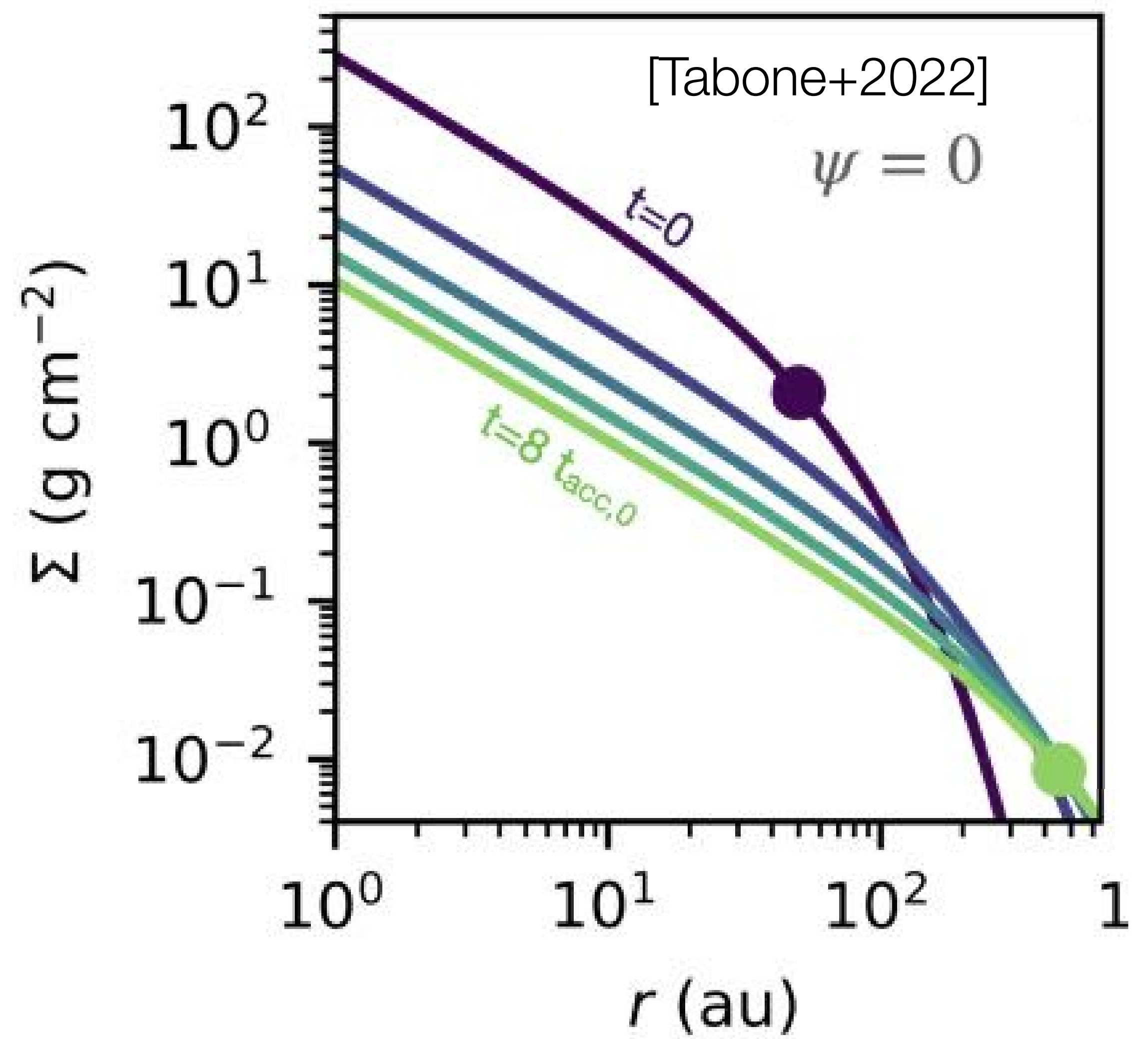


FIG. 4. *The surface density distribution with radius at four times for the δ function initial distribution of the disc. The lower dotted modification corresponds to a central star whose radius is* 10^{-2} *in these units, while the upper modification corresponds to the solution in which a no-central-flux boundary condition is imposed, corresponding to the throwing off of the disc by a strong magnetosphere.*



Viscous solution are characterised by accretion AND expansion

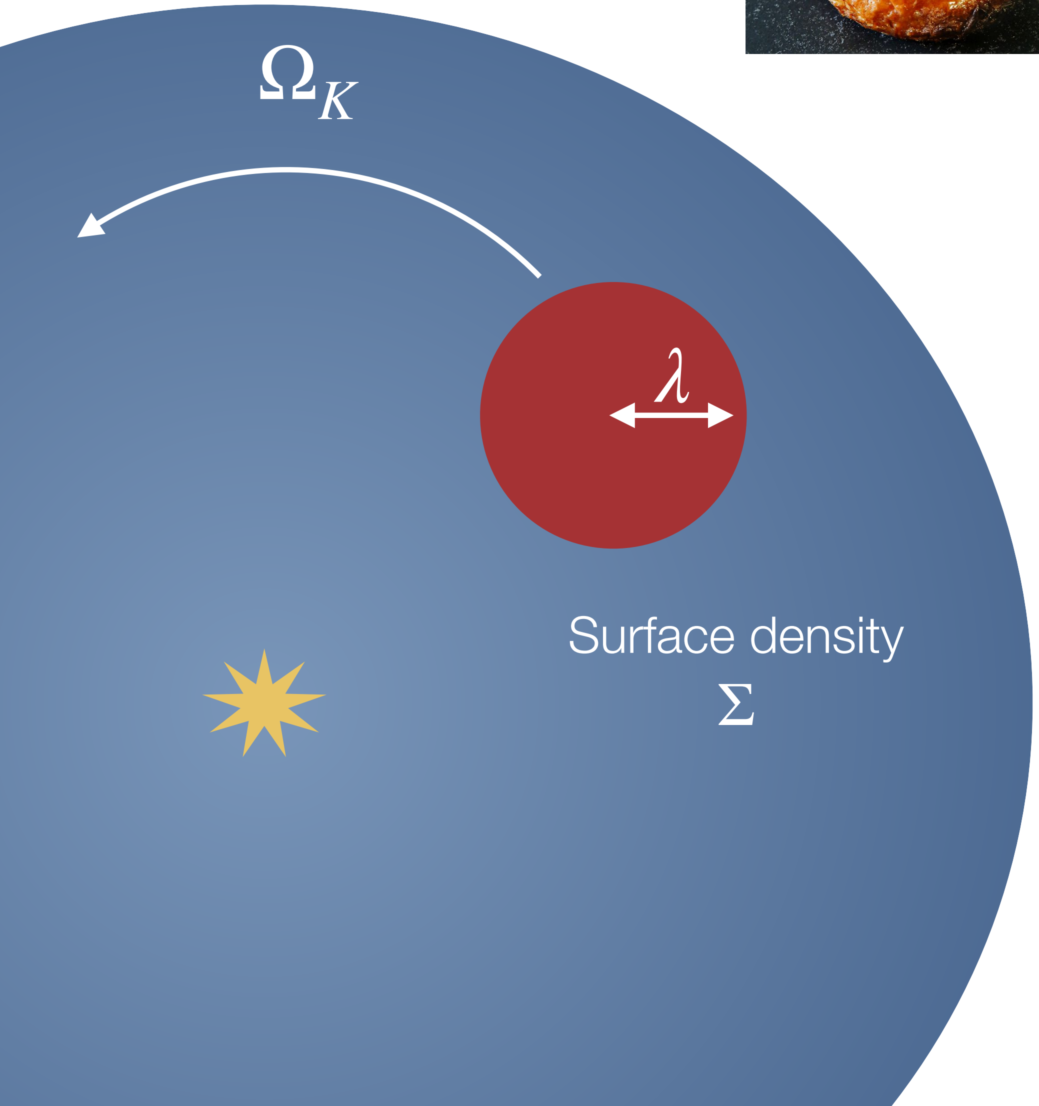
why?

Dynamics of gaseous disc

- Gravitational instability
- Vertical shear instability
- Magnetorotational instability

Gravitational instability in a nutshell

Question: when does the red blob orbiting at Ω_K collapse under its own gravity?



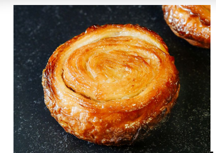
- Blob mass $M = \pi\lambda^2\Sigma$
- Free fall time of the blob
$$t_{ff} \simeq \left(\frac{\lambda^3}{GM}\right)^{1/2} = \left(\frac{\lambda}{\pi G\Sigma}\right)^{1/2}$$
- Sound crossing time
$t_{\rm sound} = \lambda / c_s$
- Orbital time
$t_{\rm orbit} = \Omega_K^{-1}$
- The blob will collapse if
$t_{ff} < t_{\rm sound}$ and $t_{ff} < t_{\rm orbit}$

Gravitational instability in a nutshell

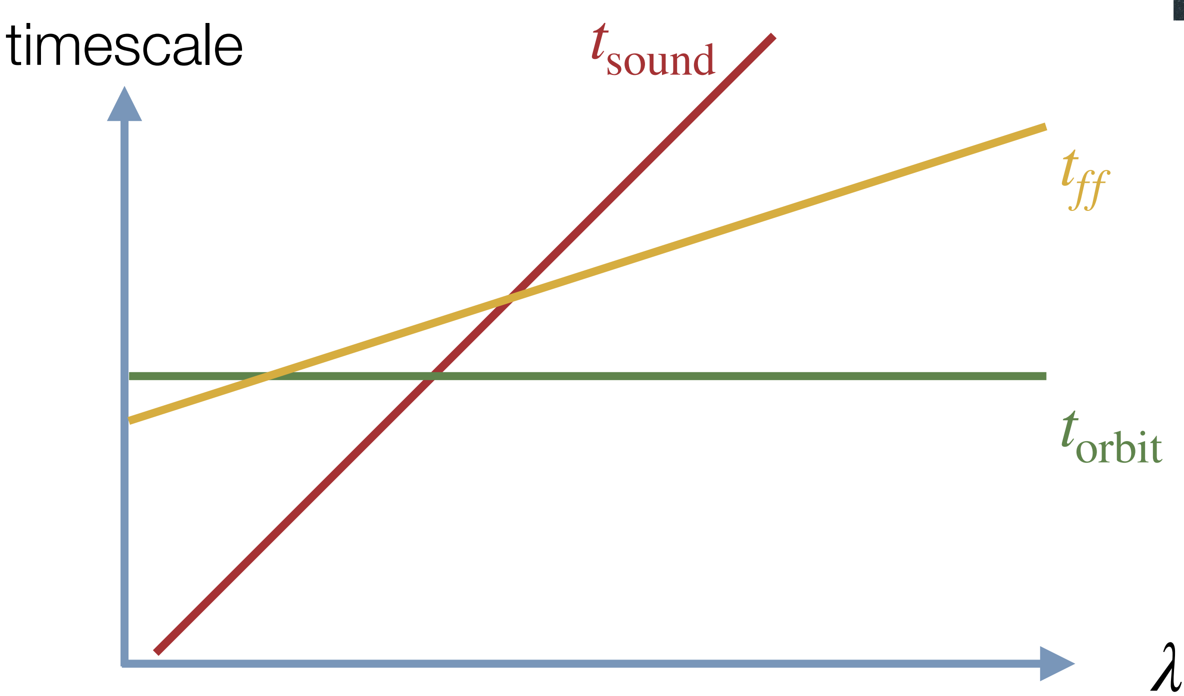
$t_{ff} = \left(\frac{\lambda}{\pi G \Sigma}\right)^{1/2}$

$t_{\rm sound} = \lambda / c_s$

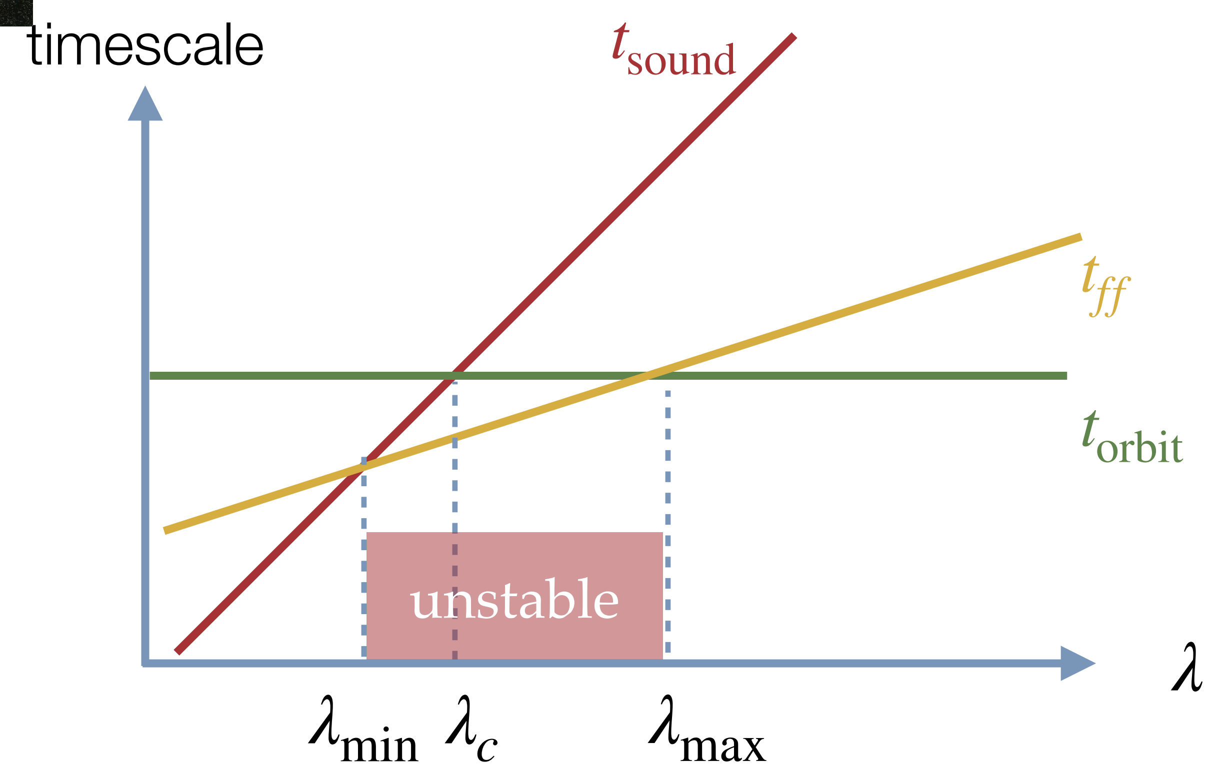
$t_{\rm orbit} = \Omega_K^{-1}$



Always stable



unstable



- Critical length scale when $t_{\rm sound} = t_{\rm orbit}$: $\lambda_c = c_s / \Omega_K$
- Unstable if $t_{ff}(\lambda_c) < t_{\rm orbit}$

$$\frac{c_s \Omega_K}{\pi G \Sigma} < 1$$

« Toomre criterion »
[A. Toomre, 1964]

« Q » parameter

Gravitational instability and disk mass

- Relate the surface density to the disk mass:
  $\Sigma \sim M_{\rm disk}/\pi R_{\rm disk}^2$

- Express the sound speed with the disk thickness
  $c_s = \Omega_K H$

- Use the Keplerian velocity definition
  $\Omega_K^2 = GM_\star/{R_{\rm disk}}^3$

$$Q = \frac{c_s \Omega_K}{\pi G \Sigma} \sim \frac{H}{R}\frac{M_{\rm disk}}{M_\star}$$

Gravitationally-unstable disks are therefore very massive (typically $M_{\rm disk} \gtrsim 0.1 M_\star$)

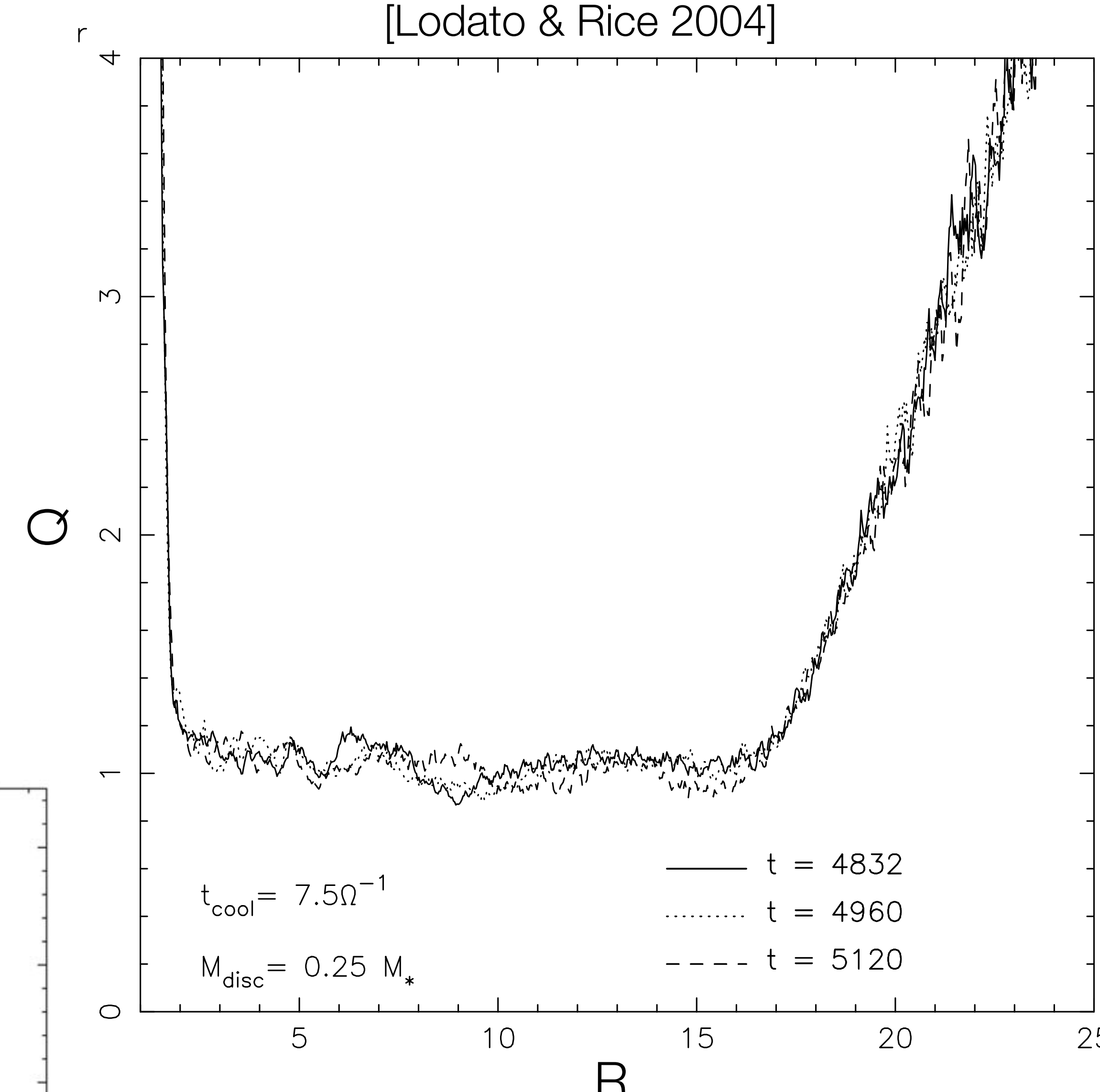
Gravitational instabilities
Nonlinear evolution

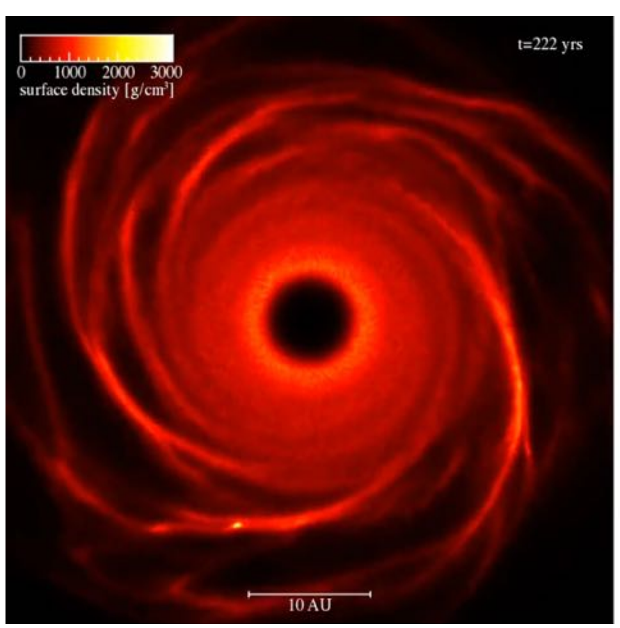
Consider a disc with Q>1

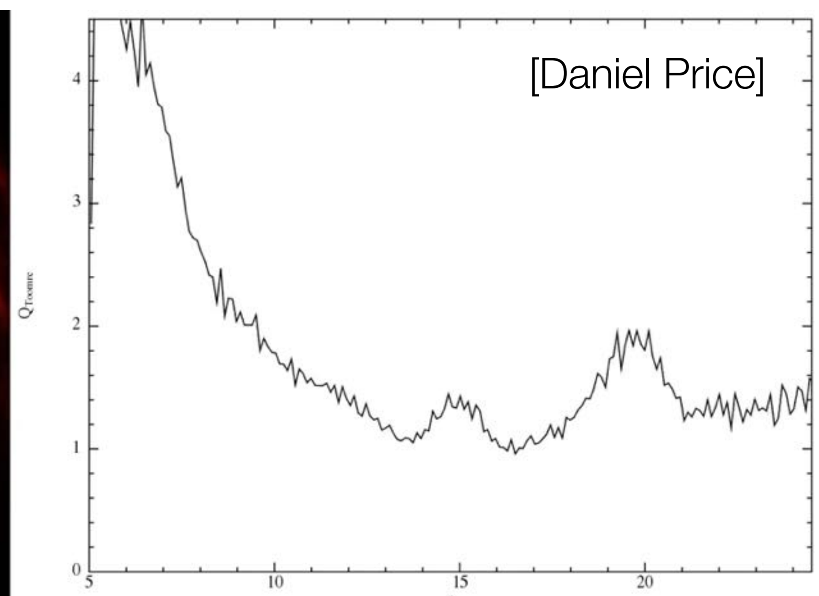
- Disc cools, cs decreases
- When Q=1 GI starts
- GI unstable modes produce strong shocks → heating
- c_s increases
- Q>1 GI stops

$$Q = \frac{c_s \Omega_K}{\pi G \Sigma}$$

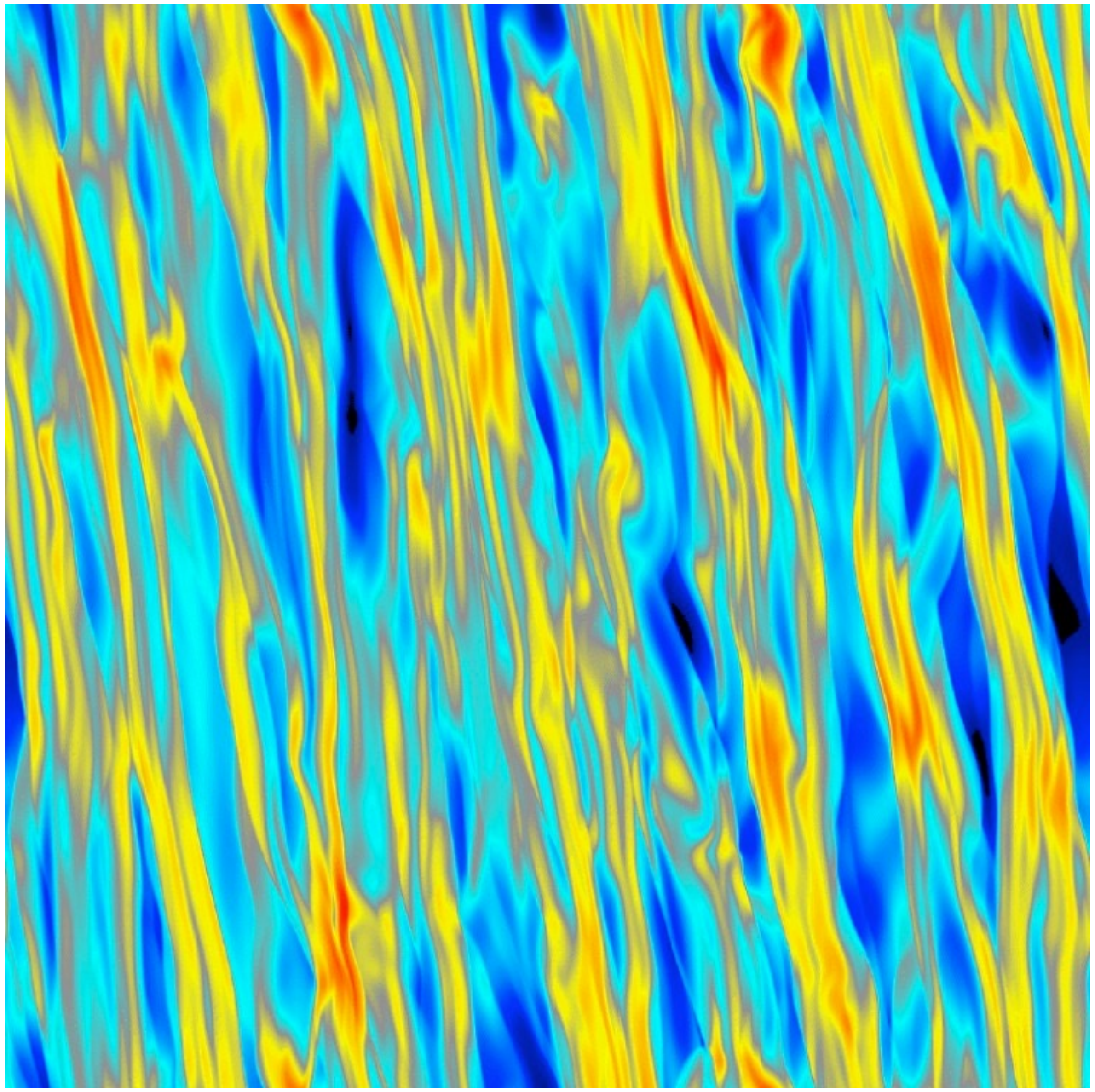
[Lodato & Rice 2004]



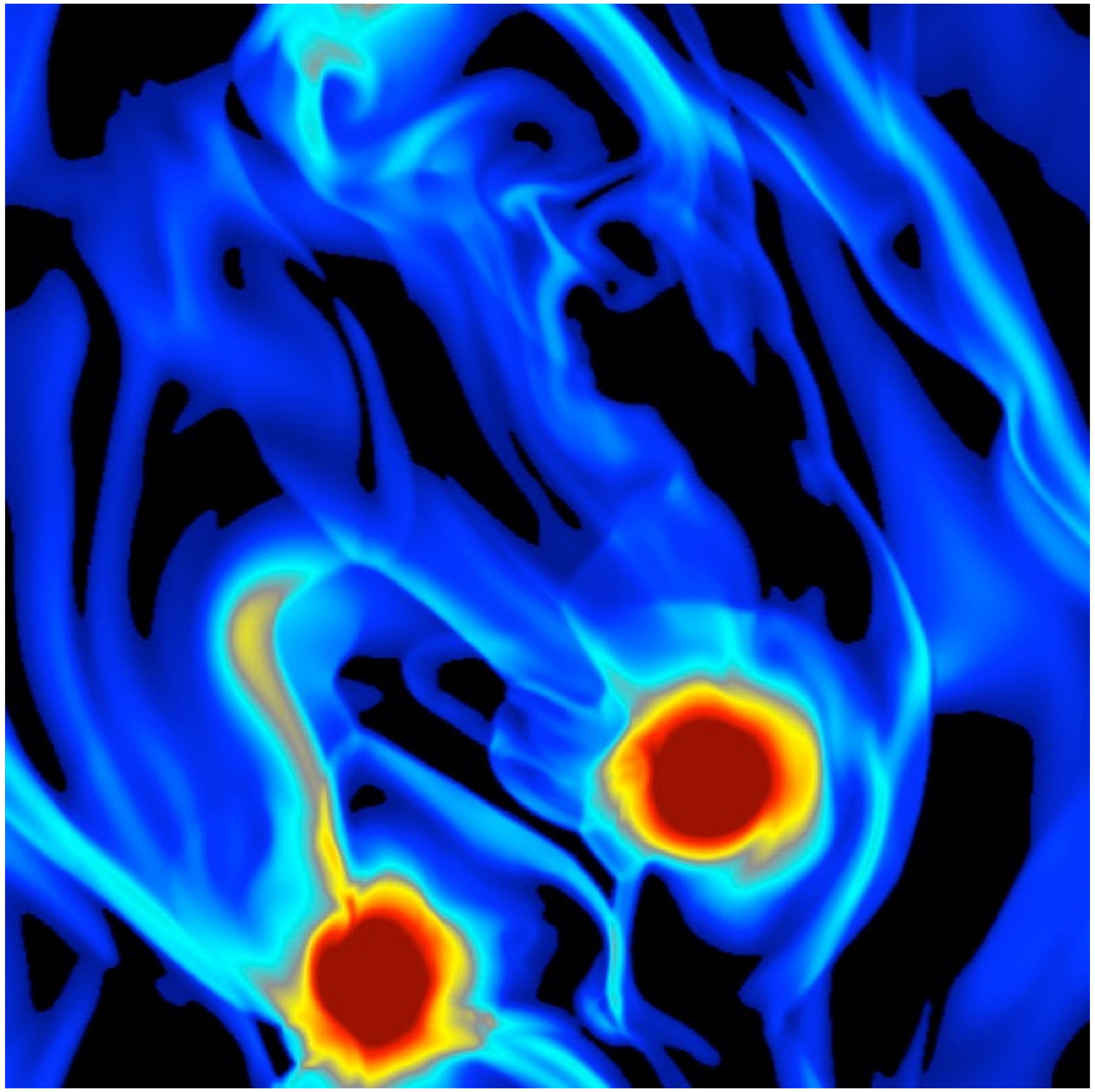




$$\tau_{\rm cool} = 10\,\Omega^{-1}$$



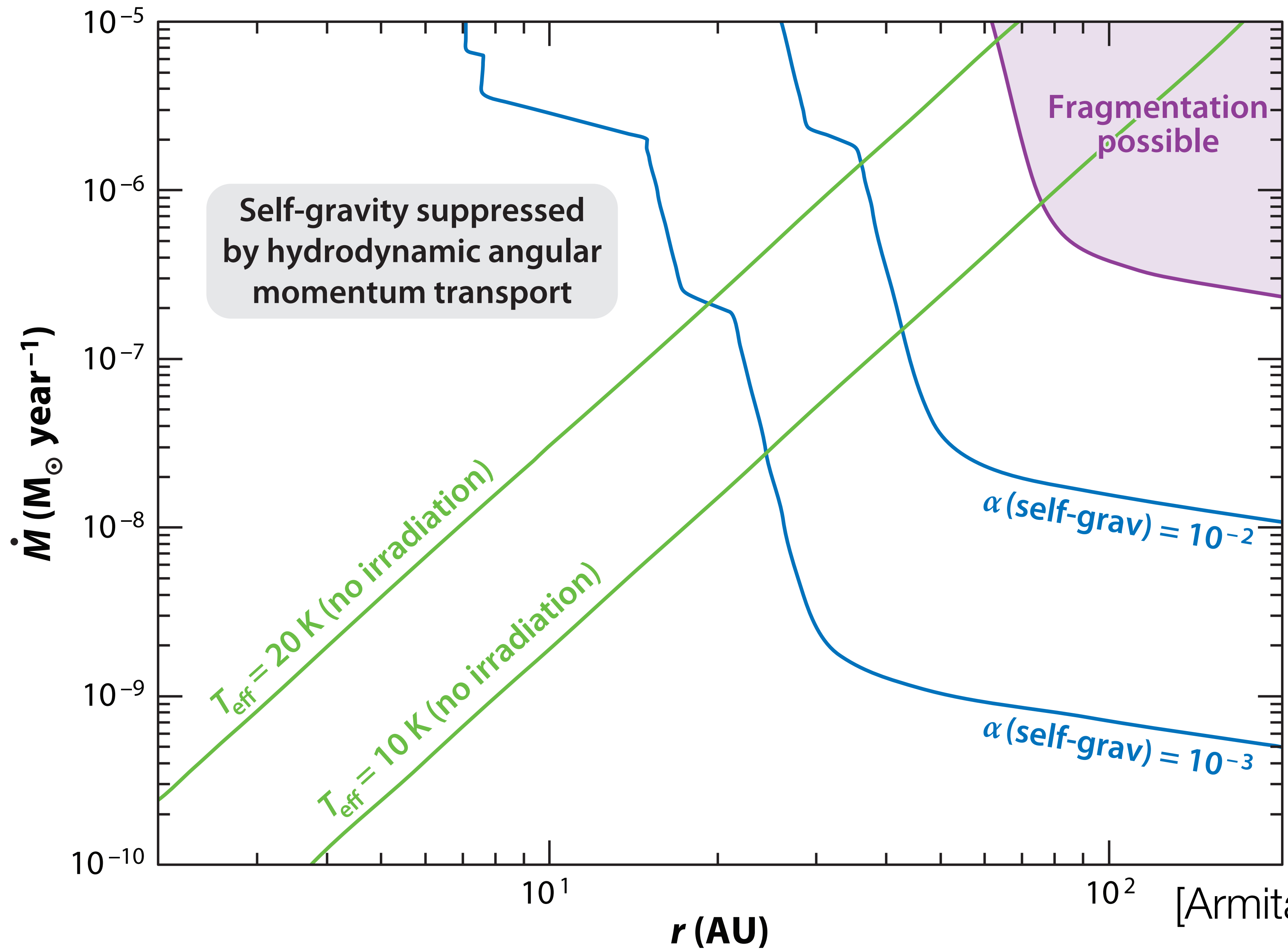
$$\tau_{\rm cool} = 2\,\Omega^{-1}$$



[Gammie 2001]

The outcome depends on the cooling timescale. If the cooling is sufficiently fast, the disc heating can't adjust, and the disk forms « clumps »

Occurence map

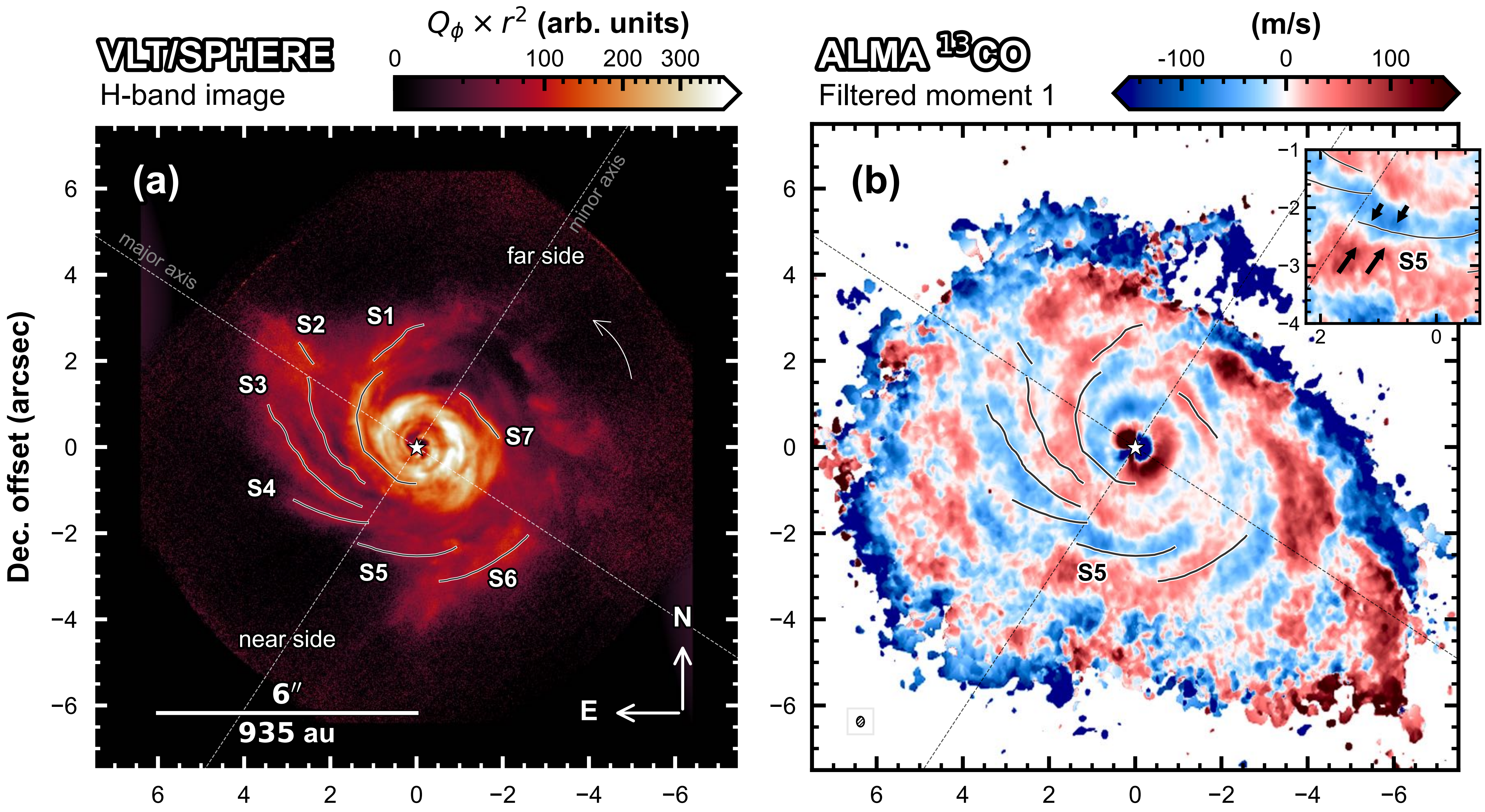


[Armitage 2011]

Expects self-gravitating discs for R>30 AU that are strongly accreting (i.e. you disks)

Gravitational instability in the literature

AB Aur



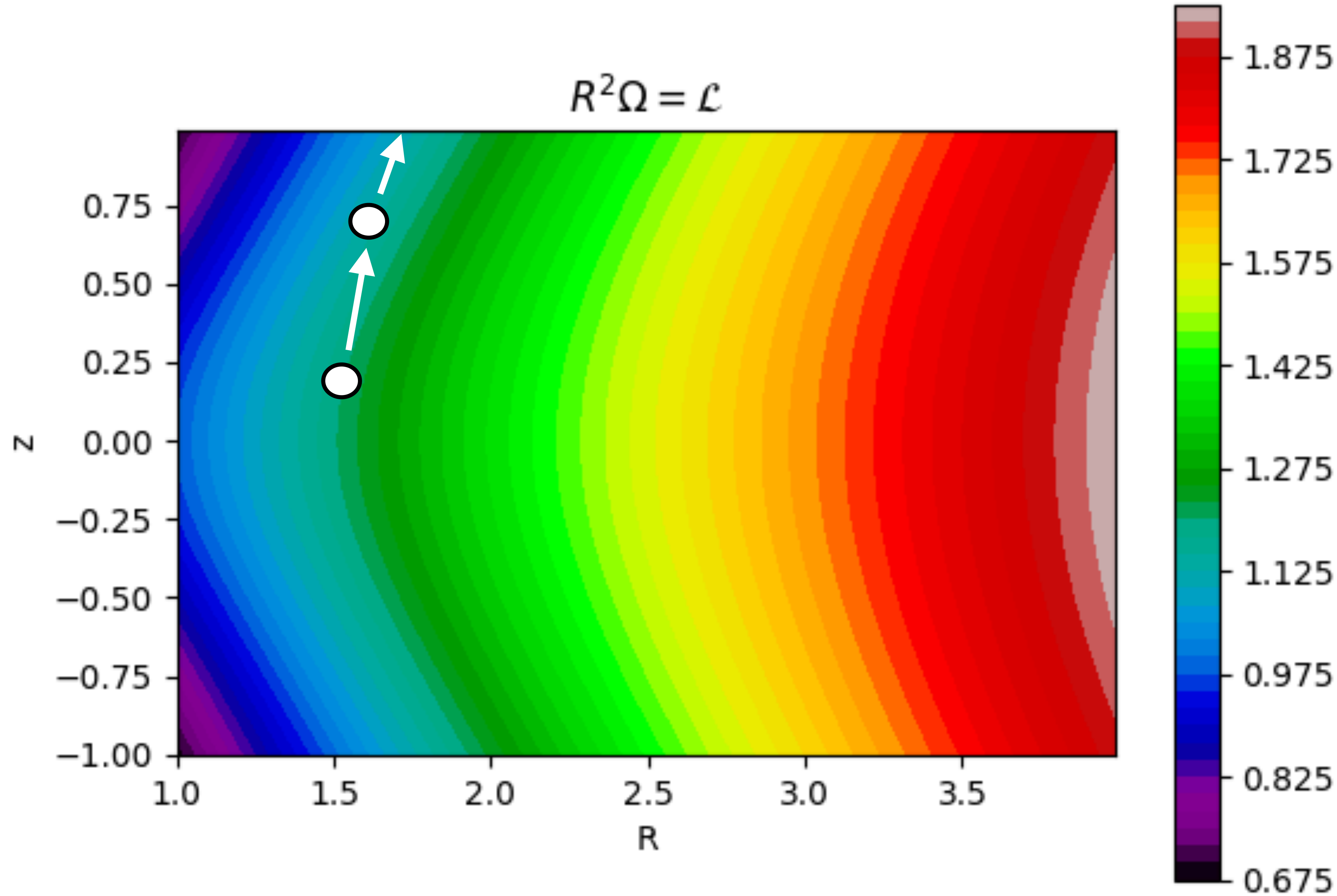
[Speedie et al., 2024, Nature]

Dynamics of gaseous disc

- Gravitational instability
- Vertical shear instability
- Magnetorotational instability

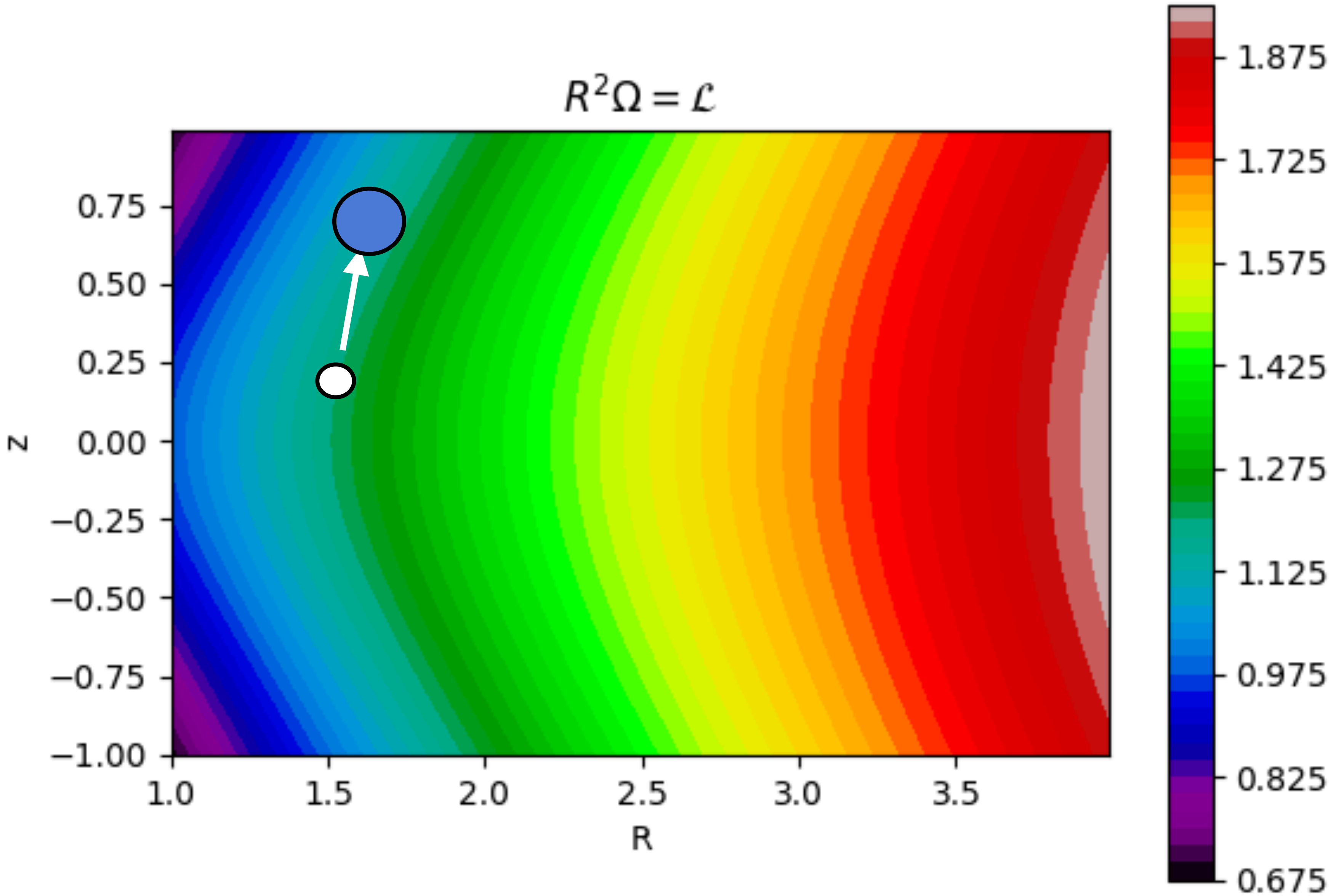
Vertical shear
A powerful source of instability

[Urpin & Brandenburg 1998
Nelson et al. 2013]



- Consider a spherical blob that we displace « almost » vertically
- The blob must conserve its specific angular momentum $\mathscr{L} = R^2\Omega$
- It ends up in a region of lower $\mathscr{L}$ → it rotates faster than the surrounding disk→it moves further out

VSI
The curse of the cooling timescale

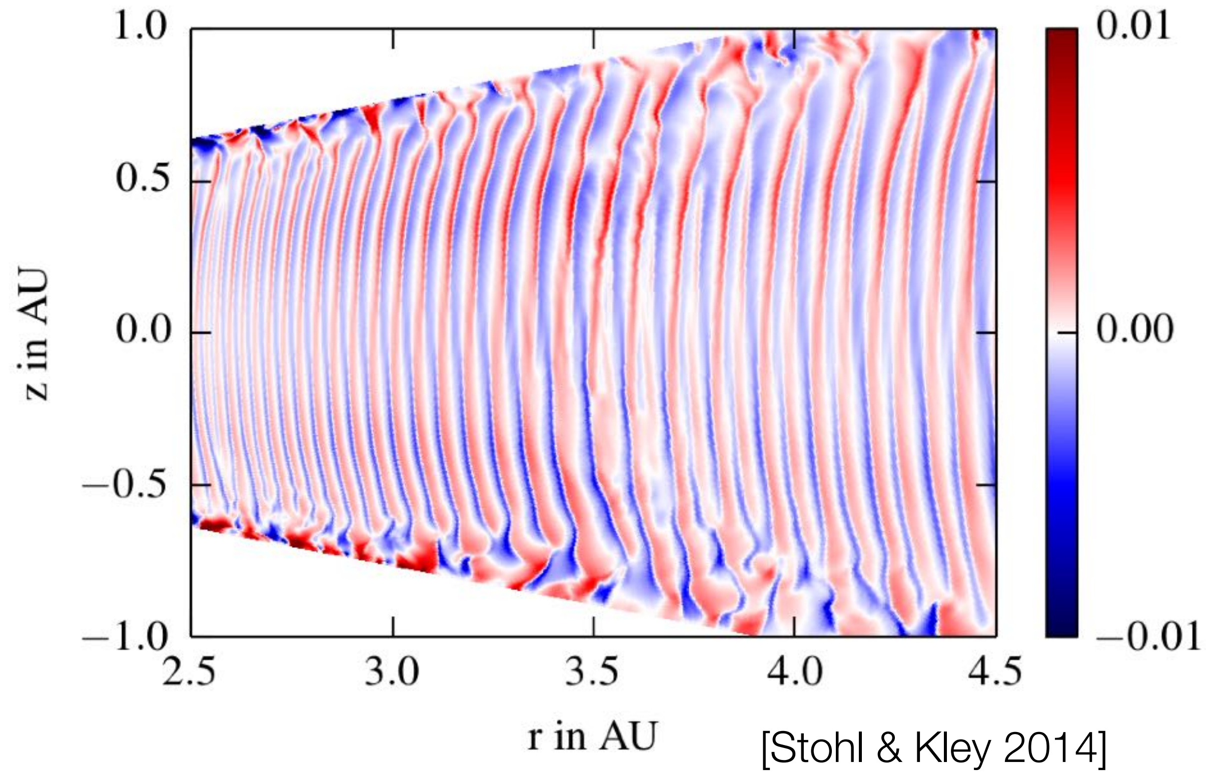


- As the particle moves up, it inflates (lower pressures!) and cools down (adiabatic expansion)
- Since the disc is vertically isothermal, the blob is cooler than the background, so it is denser
- If we don't heat up the blob *rapidly*, it comes back down because of vertical buoyancy/gravity

VSI requires a « fast » cooling/heating of the disc

VSI in practice

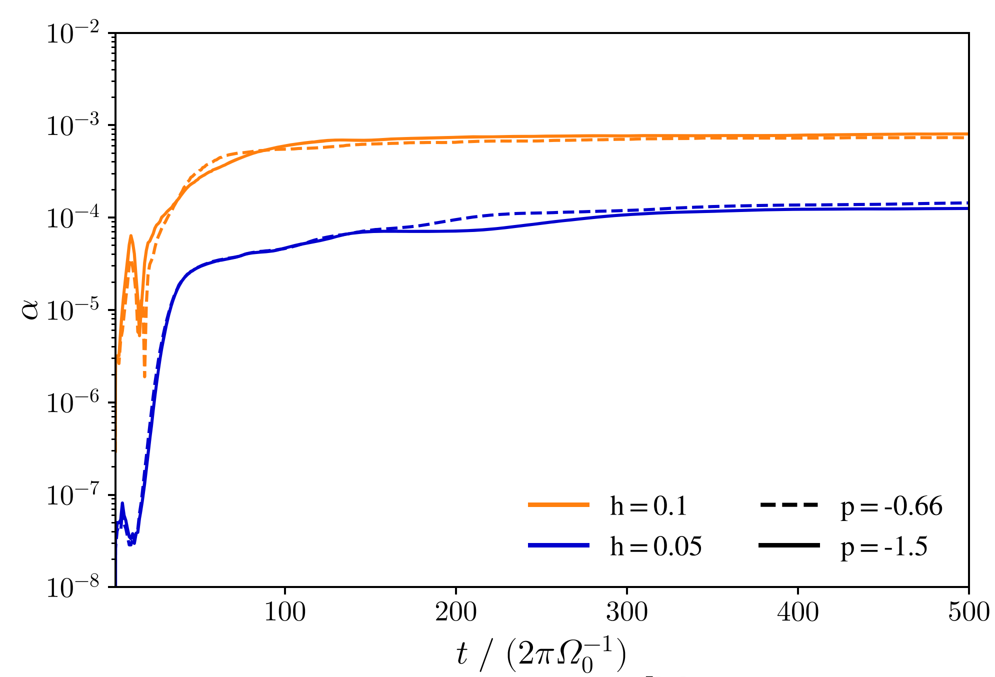
Flow topology



[Stohl & Kley 2014]

Fig. 19. Velocity in the meridional direction, u_θ, in units of local *Kepler* velocity for an irradiated run without viscosity at resolution 1024×256

Angular momentum transport



[Manger et al. 2020]

The VSI is characterised by strong up/down motions (« corrugation waves »)

Limited radial angular momentum transport ($\alpha < 10^{-3}$)

VSI in the literature

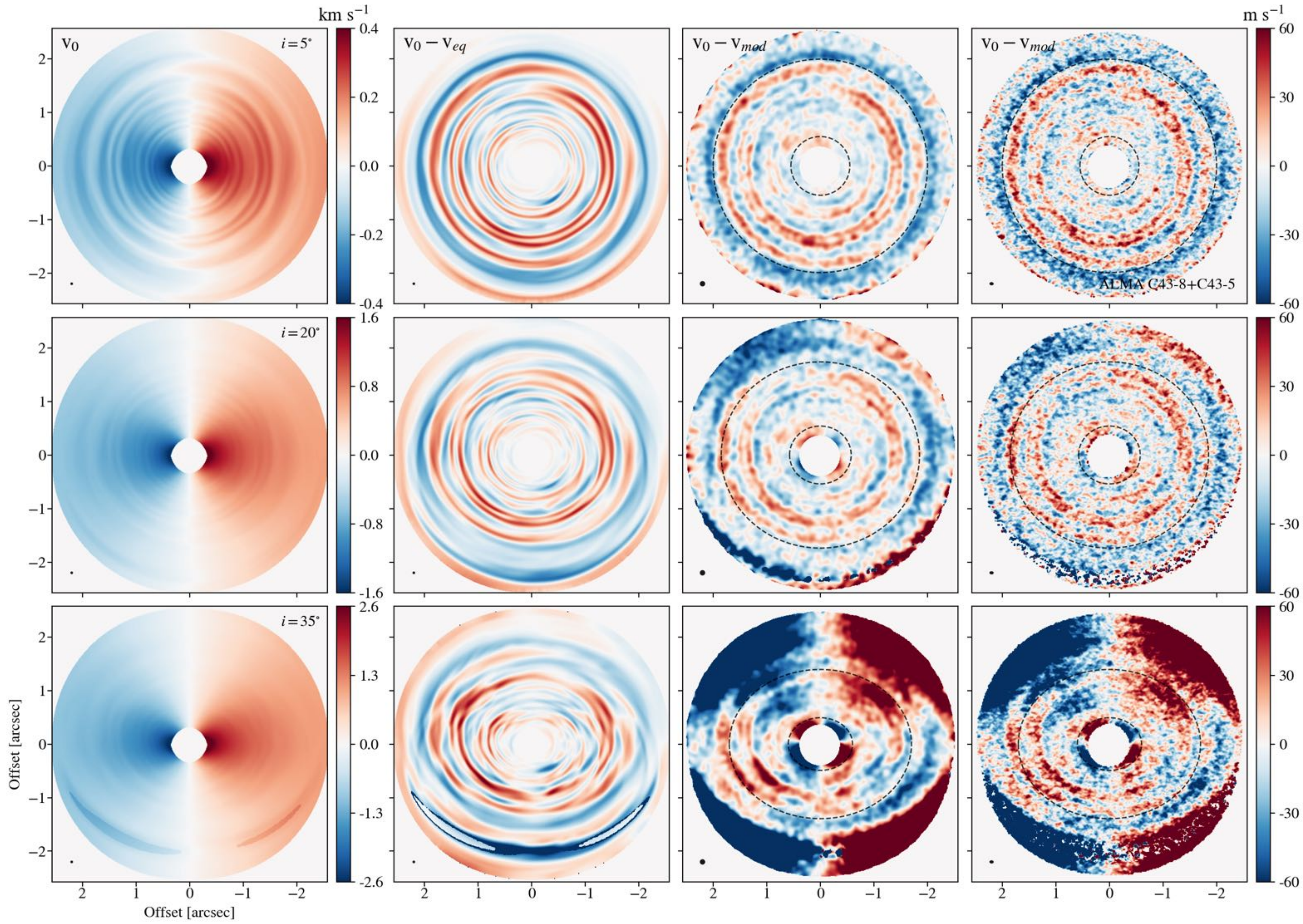


Fig. 4. Results of the line of sight velocity map and extracted velocity perturbations from a VSI unstable disk $^{12}CO(2–1)$ synthetic lines observations.

[M. Barraza-Alfaro et al. 2021]

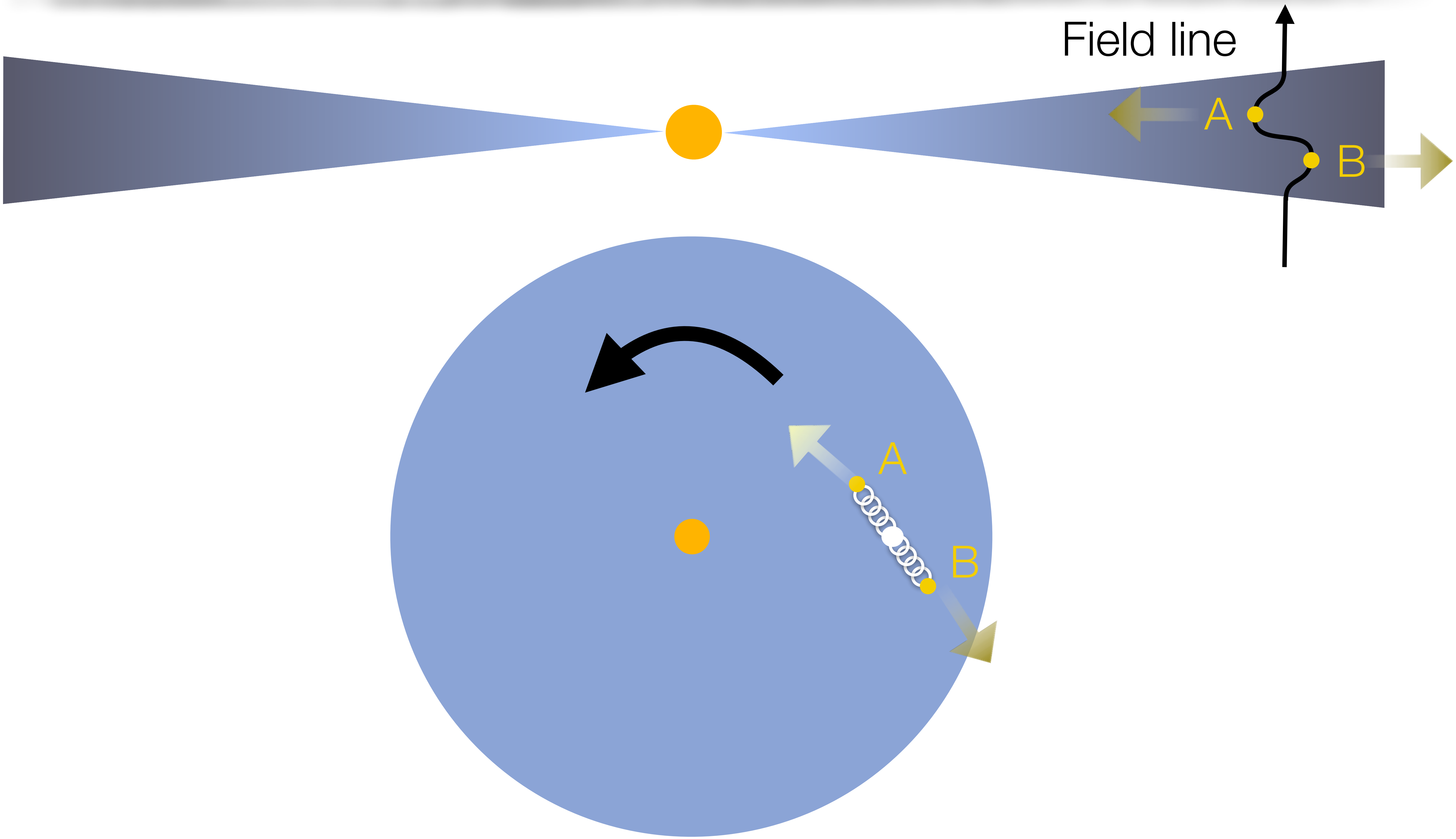
Dynamics of gaseous disc

- Gravitational instability
- Vertical shear instability
- Magnetorotational instability

Origin of turbulence in discs

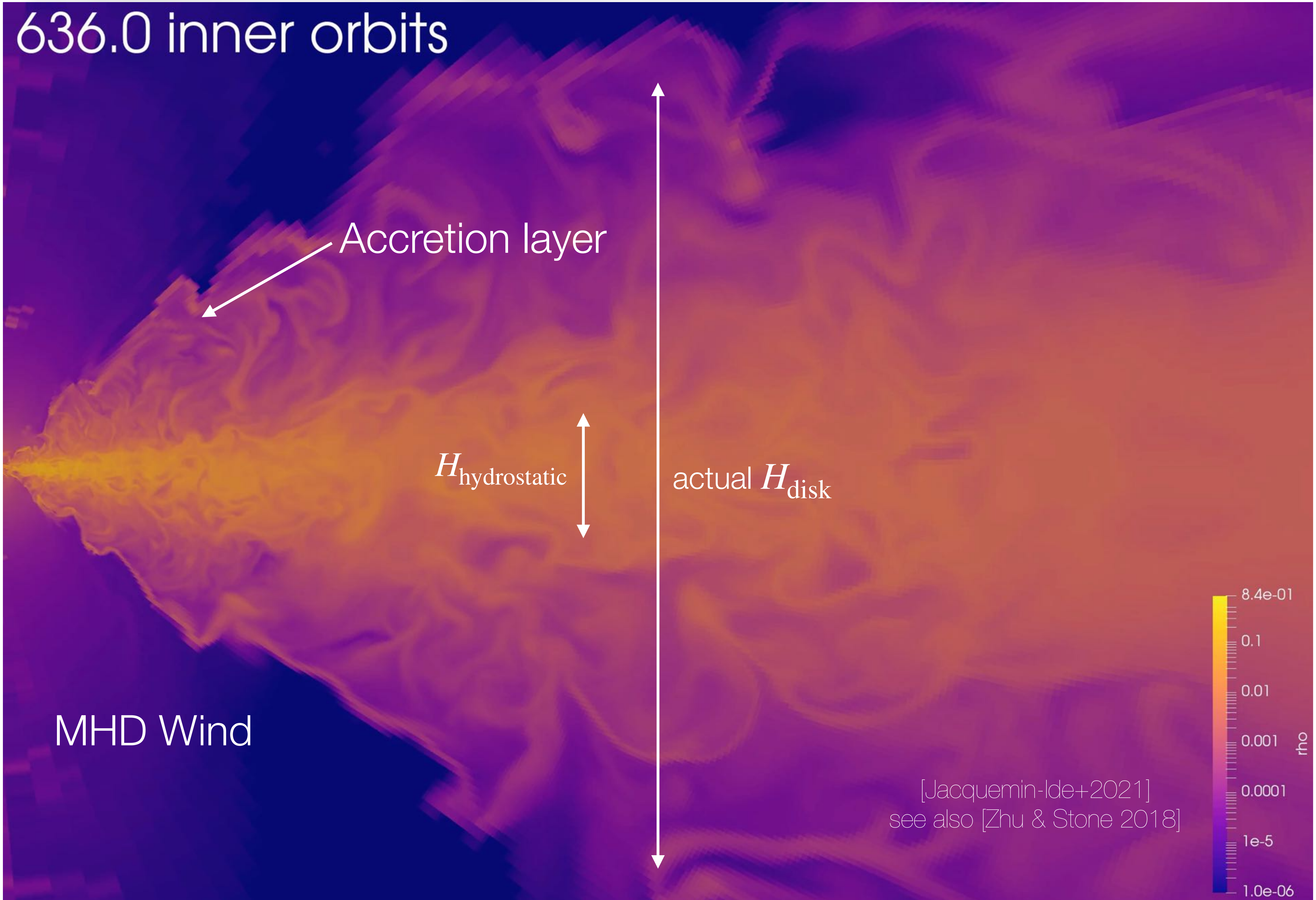
The Magnetorotational instability (MRI)

[Balbus, & Hawley (1991)]
[Balbus (2003)]



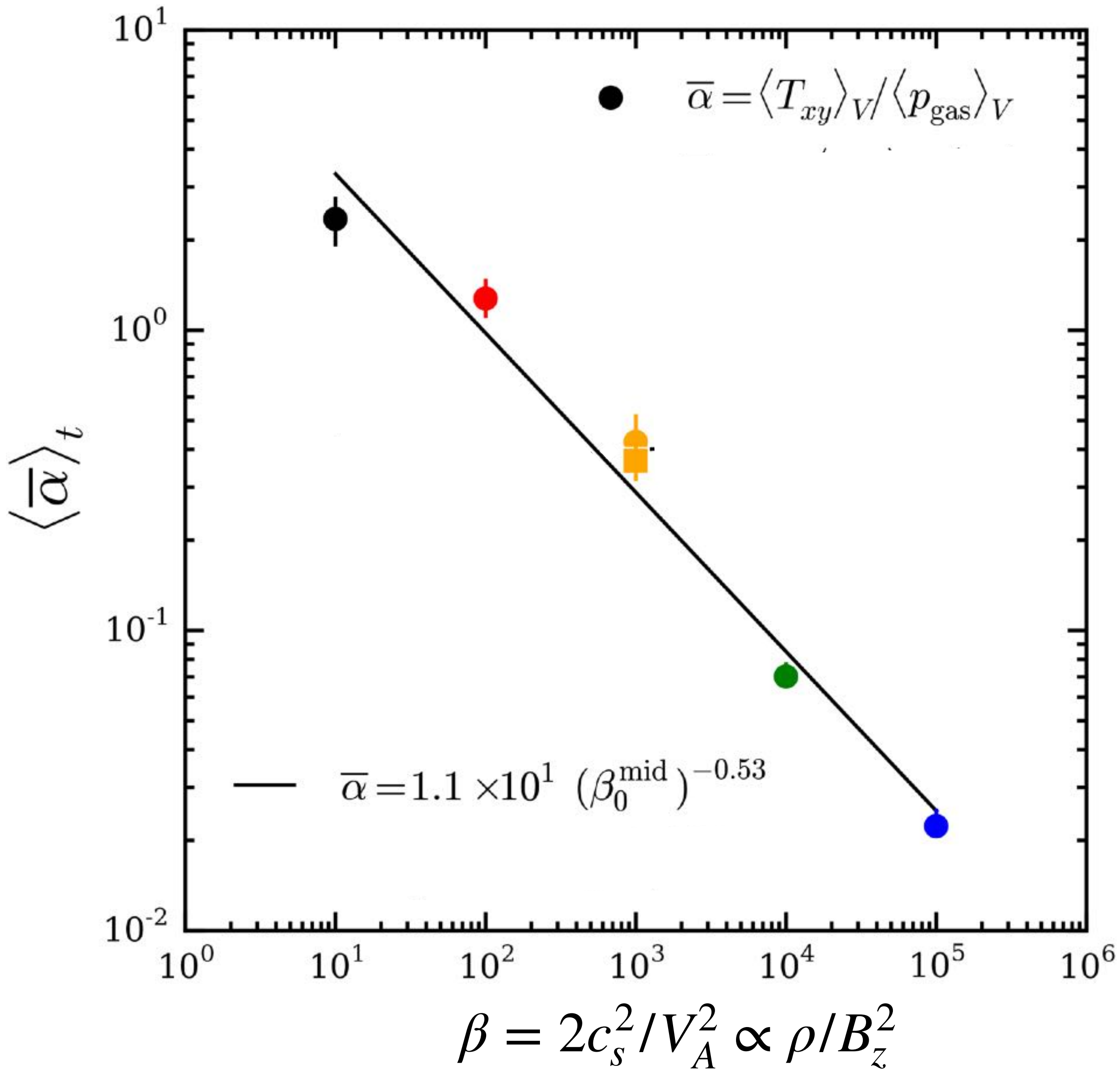
Magnetic tension between A and B transfers angular momentum between the particles and lead to a runaway

MRI-turbulent disc threaded by a large-scale B



MRI-driven angular momentum transport

[Salvesen+2016]



$\alpha_{MRI} \gtrsim 10^{-2}$ and can be $\mathcal{O}(1)$ for sufficiently strong fields

MRI: conditions of existence

1. The magnetic field must be « sufficiently » weak:

   $t_{\rm Alfven} > t_{\rm orbit} \rightarrow V_A/H < \Omega_K$

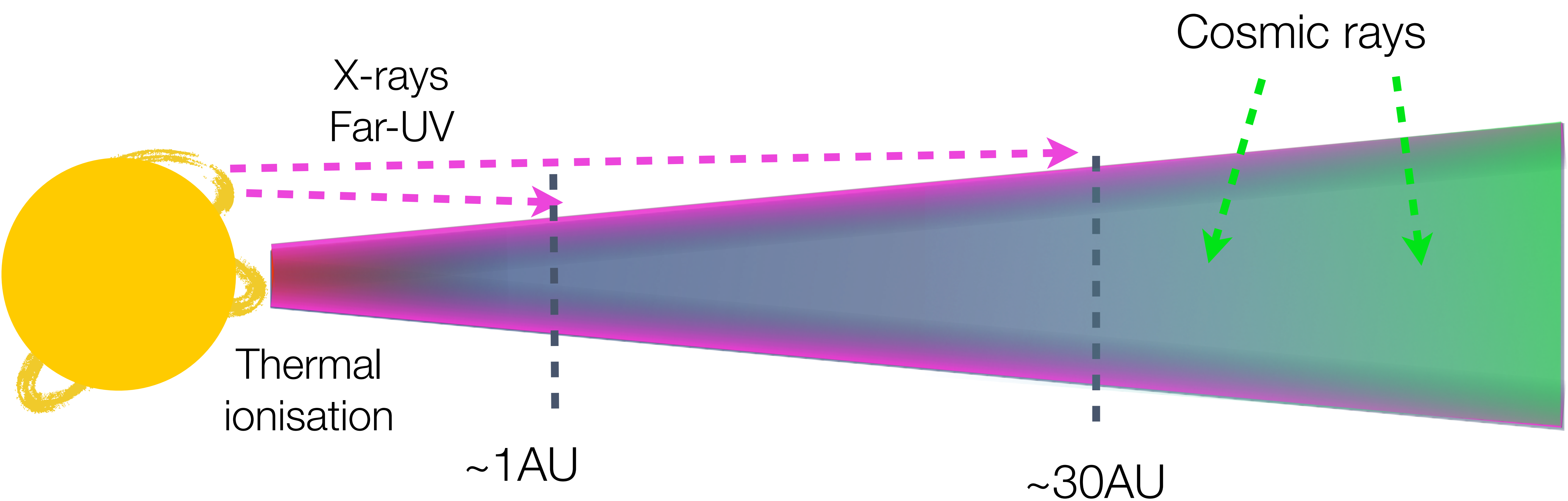
   in practice for a « standard disk »: $B < 12 R_{\rm AU}^{-11/8}\,\rm G$ [Lesur 2021]

2. The coupling between the field and the gas must be sufficiently strong

   $t_{\rm Alfven} < t_{\rm diffusion} \rightarrow V_A/H > \eta/H^2$

   In practice, there are three « kinds » of magnetic diffusivities, so this becomes a bit more complicated…

Ionisation sources in protoplanetary discs

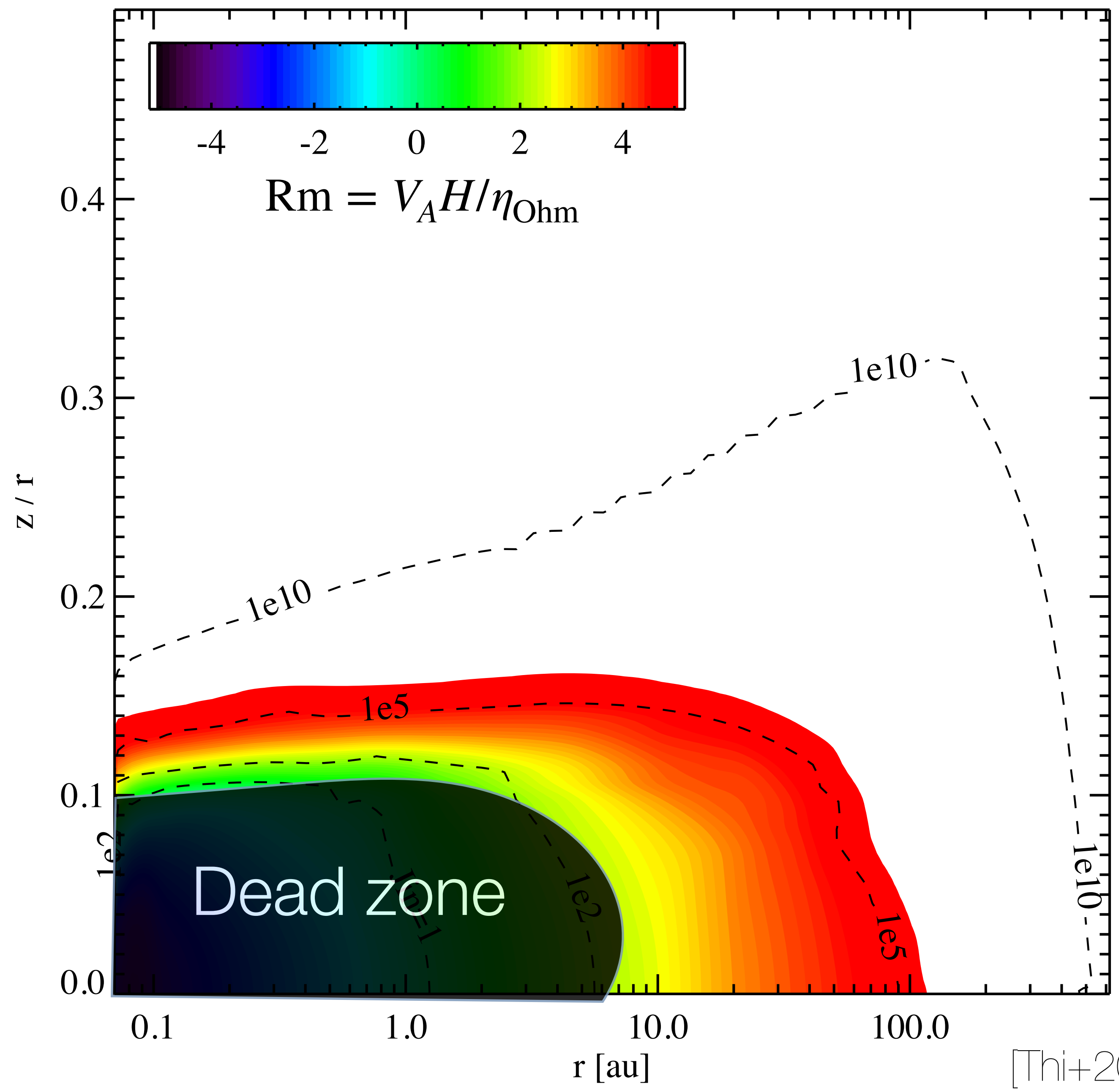


« non ideal » MHD effects

- Ohmic diffusion (electron-neutral collisions)
- Ambipolar Diffusion (ion-neutral collisions)
- Hall Effect (electron-ion drift)

Amplitude of these effects depends strongly on location & composition

Ohmic resistivity



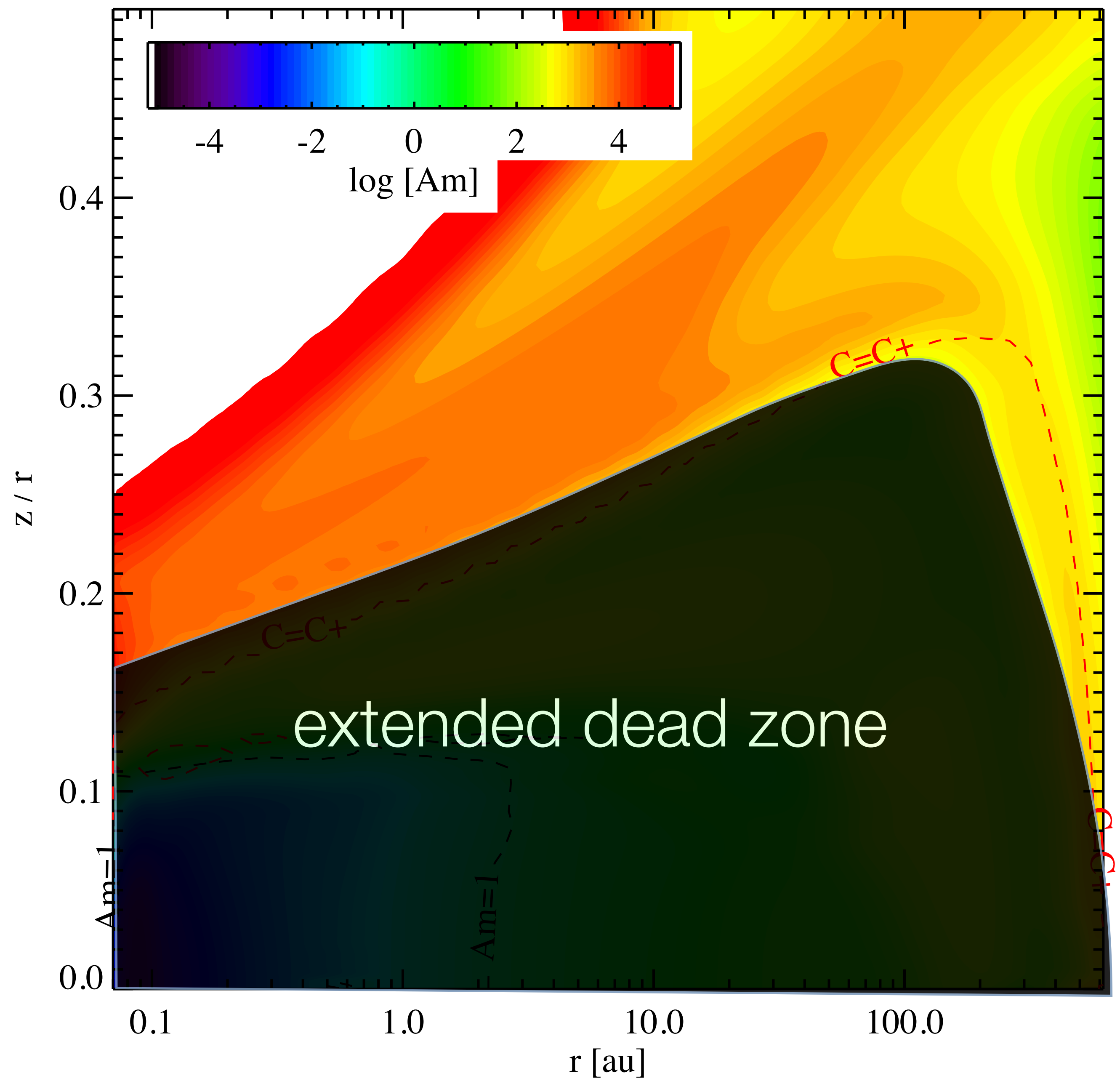
[Thi+2019]

MRI-driven turbulence is stabilised when Rm<100 [Jin 1996]

« Historical » dead zone [Gammie 1996]

Ambipolar diffusion

$$\mathrm{Am} = V_A^2/\Omega\eta_A$$

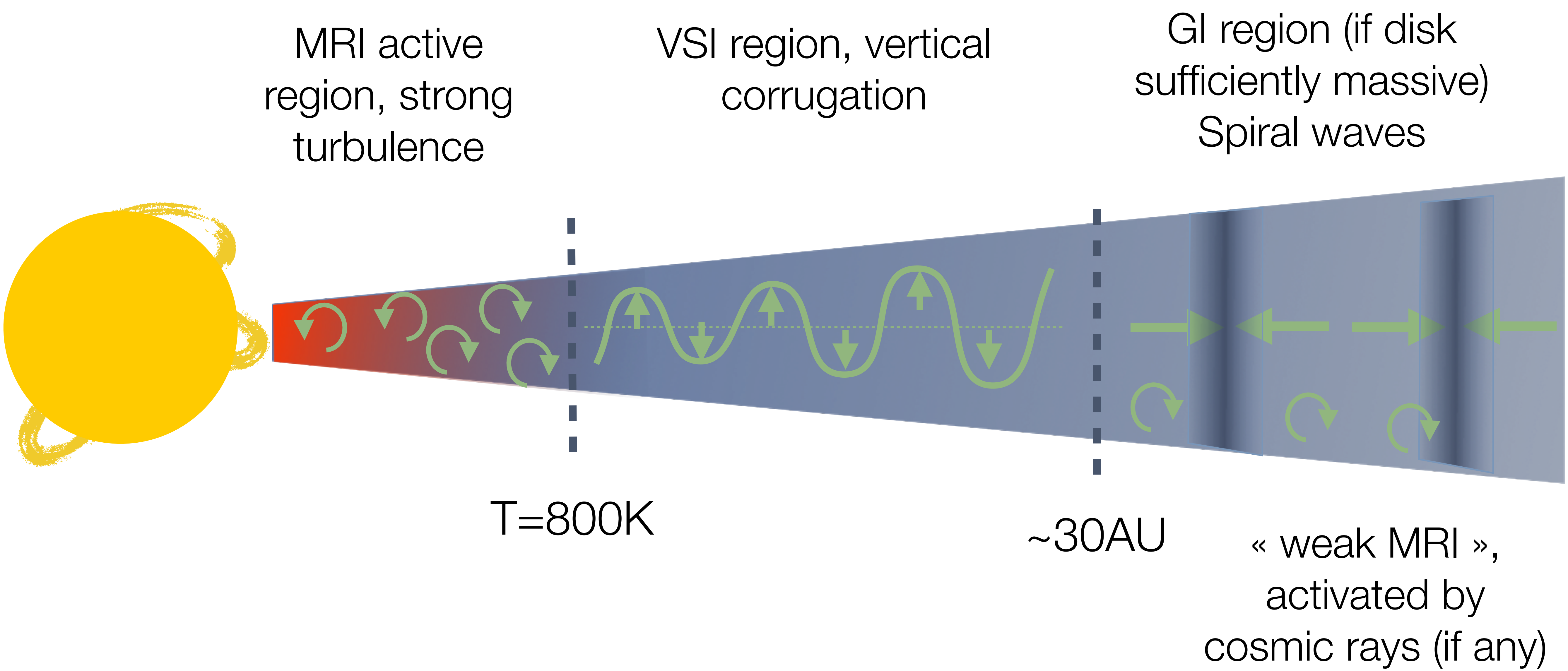


[Thi+2019]

→ Am<100 → MRI is quenched [Perez-Becker & Chiang 2011]

Discs are too diffusive to sustain MRI turbulence.

Summary



<u>this is very schematic</u>
The presence of VSI depends on the disc opacity (grain size and vertical distribution)?
GI behaviour depends on disc mass and opacity
It neglects winds (see Wednesday)